M Probability

M1

Understand and use mutually exclusive and independent events when calculating probabilities.

Link to discrete and continuous distributions.

Students should be able to:

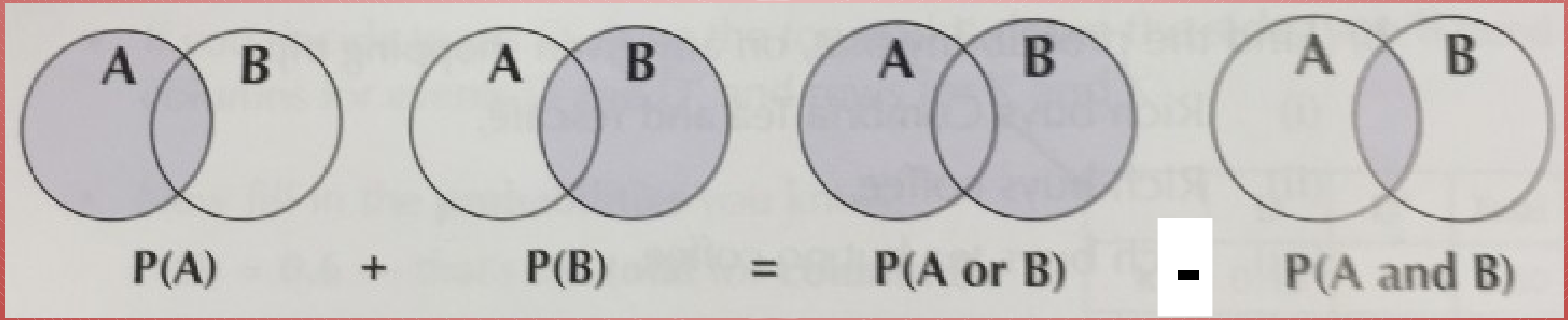
- find the probability of an event by extracting relevant information from a description of a situation (in context) or from a table of information
- recognise and use set theory notation in the context of probability, eg $\mathrm{P}(A \cup B)$, $\mathrm{P}(A \cap B)$, $\mathrm{P}(A')$
- recognise and define the meaning of mutually exclusive events, i.e. $\mathrm{P}(A \cap B) = 0$
- understand that $A \cup B$ means A or B and that, in probability, "or" is interpreted as an inclusive or, not as an exclusive or
- define the condition for two events to be independent and determine whether two events are independent by finding, and comparing, relevant probabilities, eg $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ or $\mathrm{P}(A) = \mathrm{P}(A \mid B)$, when the events A and B are independent (not required at AS).

10.1 Probability

Laws of Probability

<u>The addition law:</u>

You can see why using Venn diagrams:



10.1 Probability

Example 1a

For two events A and B, P(A B) = 0.75, P(A) = 0.45 and P(B’) = 0.4

a) Find P(A B)

P(A B) = 0.75

P(A) = 0.45

P(B) = 1 – P(B’)

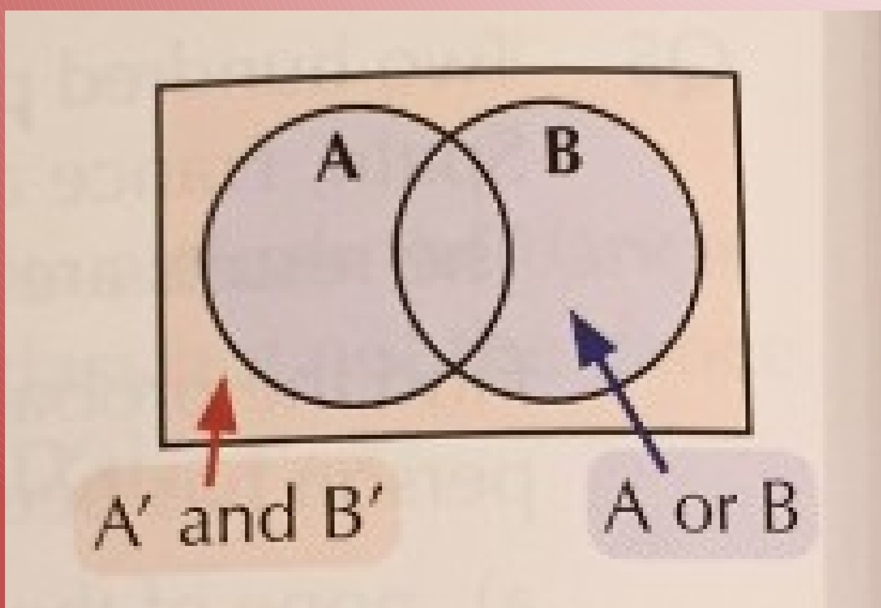
= 1 – 0.4 = 0.6

10.1 Probability

Example 1b

For two events A and B, P(A B) = 0.75, P(A) = 0.45 and P(B’) = 0.4, P(A B) = 0.3

b) Find P(A’ B’)



A’ B’ is the **complement** of A B.

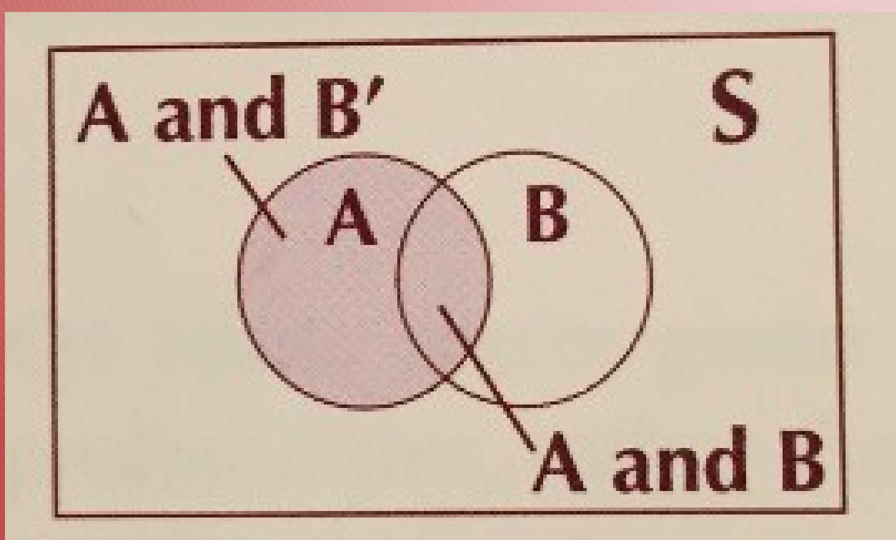
So P(A’ B’) = 1 – P(A B)
= 1 – 0.75 = **0.25**

10.1 Probability

Example 1c

For two events A and B, P(A B) = 0.75, P(A) = 0.45 and P(B’) = 0.4, P(A B) = 0.3

c) Find P(A B’)



A is made up of **A B** and **A B’**

So:

P(A B’) = P(A) – P(A B)

= 0.45 – 0.3 (from part a)

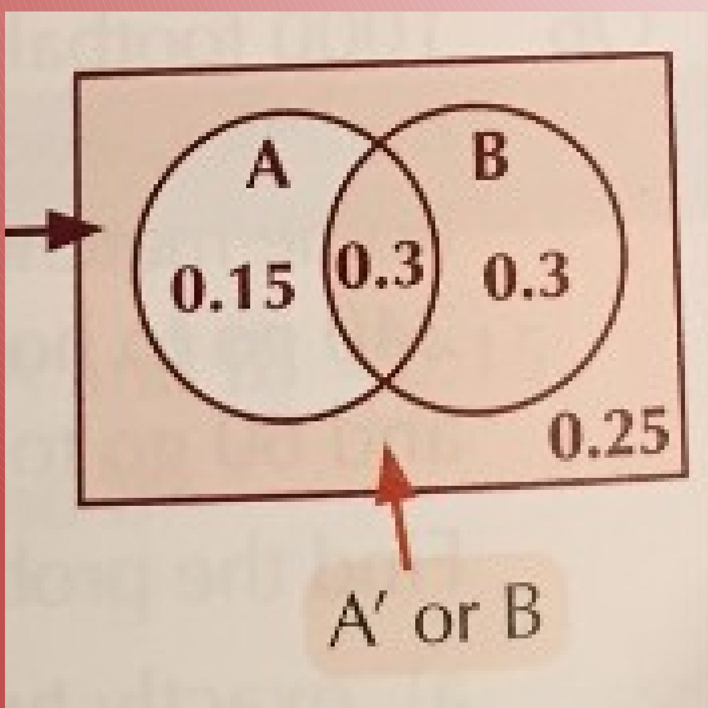
= 0.15

10.1 Probability

Example 1d

For two events A and B, P(A B) = 0.75, P(A) = 0.45 and P(B’) = 0.4 , P(A B) = 0.3

d) Find P(A’ B)



Use the probabilities given and what you’ve calculated to draw a Venn diagram.

P(A’ B) = 0.25 + 0.3 + 0.3

= 0.85

10.1 Probability

Example 2a

On any given day, the probability that Jason eats an apple is 0.6, the probability he eats a banana is 0.3, and the probability he eats both an apple and a banana is 0.2.

a) Find the probability that he eats an apple or a banana (or both)

Let A be the event ‘eats an apple’.
Let B be the event ‘eats a banana’.

We need P(A B):

10.1 Probability

Example 2b

Exercise 3.1 from sheet

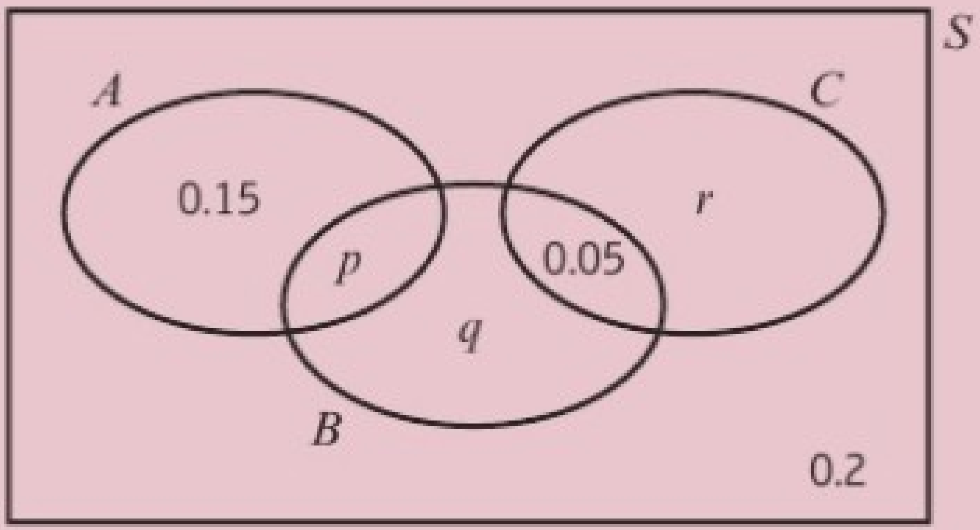
On any given day, the probability that Jason eats an apple is 0.6, the probability he eats a banana is 0.3, and the probability he eats both an apple and a banana is 0.2.

b) Find the probability that he either doesn’t eat an apple, or doesn’t eat a banana.

We want to find P(A’ B’):

Challenge

The Venn diagram shows the probabilities of a group of children liking three types of sweet.



Given that P(B) = 2P(A) and that P(not C) = 0.83, find the values of p, q and r.

Challenge

$P(B) = p + q + 0.05$

$P(A) = 0.15 + p$

As $P(B) = 2P(A)$, $p + q + 0.05 = 2(0.15 + p)$, or $p + q + 0.05 = 0.3 + 2p$

So our first equation relating p and q is: $q = 0.25 + p$

As P(not C) = 0.83

$0.15 + p + q + 0.2 = 0.83$, so our second equation results: $p + q = 0.48$

Using substitution to solve simultaneously:

$p + (0.25 + p) = 0.48$, so $2p = 0.23$ and therefore $p = 0.115$

$q = 0.25 + 0.115 = 0.365$

$P(C) = 1 – P(\text{not } C) = 1 − 0.83 = 0.17$

Hence $r + 0.05 = 0.17$, so $r = 0.12$

$p = 0.115$, $q = 0.365$, $r = 0.12$

Question 17 (+)**

The results of 100 people taking part in a wine tasting survey are shown below.

- 90 people liked wine A.
- 90 people liked wine B.
- 92 people liked wine C.
- 88 people liked wine A and B.
- 86 people liked wine B and C.
- 87 people liked wine A and C.
- 85 people liked wine A, B and C.

a) Draw a fully completed Venn diagram to represent this data.

Find the probability that a randomly chosen person …

b) … likes only two out of the three wines.

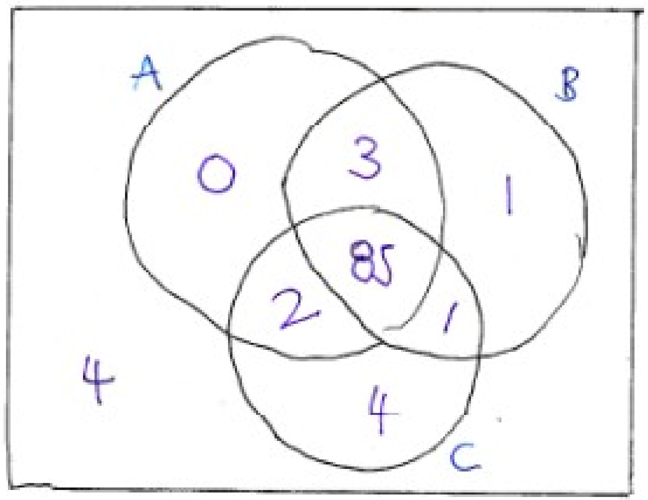
c) … likes wine B but not wine C.

d) … does not like wine B.

A person who likes at least two types of wine is selected.

e) Determine the probability that this person likes wines A and C.

a) STARTING WITH A "TRIPLE" VENN DIAGRAM



LOOKING AT THE VENN DIAGRAM

b) $P(\text{TWO OUT OF THE THREE}) = \dfrac{3+2+1}{100} = \dfrac{6}{100}$

c) $P(\text{B BUT NOT C}) = \dfrac{3+1}{100} = \dfrac{4}{100}$

d) $P(B') = \dfrac{2+4+4}{100} = \dfrac{10}{100}$

e) FINALLY THE CONDITIONAL PROBABILITY

$$P(A \cap C \mid \text{LIKES TWO}) = \frac{\frac{2}{100} + \frac{85}{100}}{\frac{3}{100} + \frac{2}{100} + \frac{1}{100} + \frac{85}{100}} = \frac{87}{91} \approx 0.956$$